Tentamen E & M 2 11-6-1997

1a) $V_A = \dfrac{j\omega L}{R + j\omega L} V_0$

b) $V_A = \dfrac{\omega L}{\sqrt{R^2+\omega^2 L^2}} V_0 \cos\left(\omega t - \arctan \dfrac{\omega L}{R}\right)$

c) $V_B = \dfrac{R V_0}{R + \frac{1}{j\omega C}} = \dfrac{j\omega R C V_0}{1 + j\omega R C}$

$\therefore V_A - V_B = \left\{ \dfrac{j\omega L}{R + j\omega L} - \dfrac{j\omega R C}{1 + j\omega R C} \right\} V_0 = \dfrac{j\omega L(1+j\omega RC) - j\omega RC(R+j\omega L)}{(R+j\omega L)(1+j\omega RC)}$

$= \dfrac{j\omega L - j\omega R^2 C}{(R+j\omega L)(1+j\omega RC)} \rightarrow V_{AB} = 0$ als $L = R^2 C \rightarrow \underline{C = \dfrac{L}{R^2}}$

d) $Z = (R + j\omega L) // \left(R + \dfrac{1}{j\omega C}\right)$

$\dfrac{1}{Z} = \dfrac{1}{R+j\omega L} + \dfrac{1}{R + \frac{1}{j\omega C}} = \dfrac{1}{R+j\omega L} + \dfrac{1}{R + \frac{R^2}{j\omega L}} = \dfrac{1}{R+j\omega L} + \dfrac{j\omega L}{R^2 + j\omega L R} =$

$= \dfrac{R + j\omega L}{R(R+j\omega L)} = \dfrac{1}{R} \rightarrow I = \dfrac{V_0}{R}$

2a) $\dfrac{1}{C} = \dfrac{1}{C_1} + \dfrac{1}{C_2}$ met $C_1 = \dfrac{2\varepsilon_0 \varepsilon_r A}{d}$ en $C_2 = \dfrac{2\varepsilon_0 A}{d}$

$\therefore \dfrac{1}{C} = \dfrac{d}{2\varepsilon_0\varepsilon_r A} + \dfrac{d}{2\varepsilon_0 A} = \dfrac{d(\varepsilon_r + 1)}{2\varepsilon_0\varepsilon_r A} \rightarrow C = \dfrac{2\varepsilon_0\varepsilon_r A}{d(\varepsilon_r+1)}$

b) $D_{1n} - D_{2n} = \sigma_c$ Hier: $\sigma_c = 0 \rightarrow D_{1n} = D_{2n}$

c) $V = -\int \vec{E} \cdot d\vec{l} = -E_1 \cdot \dfrac{d}{2} - E_2 \dfrac{d}{2}$

$D_{1n} = D_{2n} \rightarrow \varepsilon_r E_1 = E_2$

$-E_1 \dfrac{d}{2} - \varepsilon_r E_1 \dfrac{d}{2} \rightarrow E_1 = -\dfrac{2V}{d(\varepsilon_r+1)}$

− teken: dwz; gericht van de plaat met de hoogste spanning naar de plaat met de laagste spanning

d) $P = \varepsilon_0(\varepsilon_r - 1) E_1$ (in diëlektricum)

$P = \sigma_p$

$\sigma_p = \varepsilon_0(\varepsilon_r - 1)\left(-\dfrac{2V}{d(\varepsilon_r+1)}\right) =$

$$\boxed{\sigma_p = -\dfrac{2\varepsilon_0 V}{d} \dfrac{\varepsilon_r - 1}{\varepsilon_r + 1}}$$

3a) $H_{boring} = \frac{B_x}{\mu_0} = \frac{0{,}003\ T}{1{,}26\times10^{-6}\ N/A^2} = 2380\ A/m$

$H_{yzer} = H_{boring} \quad (H_{1t} = H_{2t})$

$\Big\}\ H_{yzer} = 2380\ A/m$

b) $B_{yzer} = B_{gap} = 1{,}2\,T \qquad (B_{1n} = B_{2n})$

c) $H_{gap} = \frac{B_{gap}}{\mu_0} = 9{,}52\times10^{5}\ A/m$

d) $\oint \vec{H}.d\vec{L} = NI \quad \rightarrow \quad I = \frac{1}{N}\left\{ H_{yzer}(2\pi a - d) + H_{gap}\, d \right\} =$

$= 10^{-3}\left\{ 2380\,(0{,}312) + 9{,}52\times10^{5}\times2\times10^{-3} \right\} =$

$= 10^{-3}\left\{ 742 + 1904 \right\} = 2{,}65\,A$

e) $M = \frac{B}{\mu_0} - H_{yzer} = (9{,}52\times10^{5} - 2380) = 9{,}5\times10^{5}\ A/m$

f) $B = \mu_0\mu_r H_{yzer} \rightarrow \mu_r = \frac{B}{\mu_0 H_{yzer}} = \frac{9{,}52\times10^{5}}{2380} = 400$

4a) $\vec{\nabla}\cdot\vec{D} = \rho_c \qquad \vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$

$\vec{\nabla}\cdot\vec{B} = 0 \qquad \vec{\nabla}\times\vec{H} = \vec{J}_c + \frac{\partial\vec{D}}{\partial t}$

In vacuum: $\rho_c = 0$; $\vec{J}_c = 0$; $\vec{D} = \varepsilon_0\vec{E}$; $\vec{B} = \mu_0\vec{H}$;

$\vec{\nabla}\vec{E} = 0$; $\vec{\nabla}.\vec{B} = 0$; $\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$; $\vec{\nabla}\times\vec{B} = \varepsilon_0\mu_0\frac{\partial\vec{E}}{\partial t}$

b) Vlakke golf (in y-z vlak): $\vec{E} = \vec{E}(x,t) \quad \vec{B} = \vec{B}(x,t) \rightarrow$

$\frac{\partial\vec{E}}{\partial y} = 0$; $\frac{\partial\vec{E}}{\partial z} = 0$; $\frac{\partial\vec{B}}{\partial y} = 0$; $\frac{\partial\vec{B}}{\partial z} = 0$;

$\vec{\nabla}.\vec{E} = 0 \rightarrow \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 0 \rightarrow \frac{\partial E_x}{\partial x} = 0$

$\vec{\nabla}.\vec{B} = 0 \rightarrow \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = 0 - \frac{\partial B_x}{\partial x} = 0$

$\Big\} \rightarrow$ vlakke golf is transversaal

c) $\vec{N} = \vec{E}\times\vec{H}$; elektromagn. energie die per tijdseenheid door een eenheidsoppervlak stroomt.

d) $\vec{N} = \frac{1}{\mu_0}(\vec{E}\times\vec{B}) = \frac{1}{\mu_0}\left\{(E_yB_z - E_zB_y)\hat{x} + (E_zB_x - E_xB_z)\hat{y} + (E_xB_y - E_yB_x)\hat{z}\right\} =$

$= \frac{1}{\mu_0}\left\{E_xB_y - E_yB_x\right\}\hat{z} = \frac{1}{\mu_0}\left\{a_1b_2 - a_2b_1\right\}\sin^2(kz - \omega t)\,\hat{z}$